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TURBULENCE INTENSITY IN DILUTE TWO-PHASE FLOWS—3

THE PARTICLES–TURBULENCE INTERACTION IN DILUTE TWO-PHASE FLOW

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Abstract—We propose a simplified theory for the particles–turbulence interaction in a dilute two-phase flow with particles of arbitrary sizes. The theory takes into account two sources of turbulence in particle-laden flows: (i) the carrier fluid velocity gradients; and (ii) turbulent wakes behind the coarse particles. The theoretical description is based on the modified mixing-length theory and turbulent kinetic energy balance method. The solution of the problem for particle-laden flow does not require any additional quantitative empirical data; only the standard semi-empirical parameters for pure carrier fluid are used. The dimensional analysis of the system of equations is used to reduce to the minimum the number of nondimensional parameters in turbulent particle-laden flows. In the limits of fine and coarse particles the asymptotic expression for turbulence intensity is found. It is shown that in the former case the carrier fluid fluctuations intensity is found. It is shown that in the former case the carrier fluid fluctuations depend only on the value of the total mass content of the admixture; whereas in the latter one they are determined by the total mass content of the admixture, the density ratio of the phase and the aerodynamic properties of the particles. The proposed theory is applied to predict the fluctuations intensity in various types of turbulent particle-laden flows. The results of the calculations are in fairly good agreement with the experimental data.

Key Words: particles, two-phase flow, particle–turbulence interaction, turbulent fluctuations

INTRODUCTION

The particles–turbulence interaction is of great importance in understanding the mechanism of turbulence generation and transfer in two-phase flows. This problem is also important for developing calculational methods for devices which use multiphase mixtures. During the last decade this problem has been the motivation for a number of investigations covering various aspects of this phenomenon (e.g. Hetsroni 1989; Gore & Crowe 1989; Rashidi *et al.* 1990, 1991). The following situations have been studied experimentally: particle-laden turbulent jets (Hetsroni & Sokolov 1971; Laats & Frishman 1973; Parthasarathy & Faeth 1987; Shuen *et al.* 1985); two-phase pipe flows (Tsuji & Morikawa 1982; Tsuji *et al.* 1984; Zisselmar & Molerus 1979; Maeda *et al.* 1980; Alajbegovic *et al.* 1994); two-phase boundary layers (Rashidi *et al.* 1990, 1991); and two-phase homogeneous turbulence (Parthasarathy & Faeth 1987, 1990).

In the early investigations on particle-laden turbulent jets it was discovered that the presence of solid particles leads to the modulation of the turbulence of the carrier fluid (Hetsroni & Sokolov 1971; Laats & Frishman 1973). This effect manifests itself not only in the modulation of the turbulence intensity but also in changing the energy spectra—mostly a descrease in the spectral components at high-frequency and a corresponding change in the distribution of the fluctuations energy. The subsequent measurements (Shuen *et al.* 1985; Parthasarathy & Faeth 1987; Tsuji *et al.* 1988) made for wide variations in particle sizes, the phase-density ratio, the total mass content etc. corroborated these results. Incidentally, a number of new effects reflecting specific features of the particles–turbulence interaction have been discovered. It was shown (Tsuji & Morikawa 1982; Tsuji *et al.* 1984; Lee & Durst 1982) that carrier fluid turbulence intensity in particle-laden flows with coarse particles does not always decrease; the turbulence intensity may also increase. That phenomenon has a common character and may be observed in various types of particle-laden flows: jets, pipe flows, flows in the boundary layer and in homogeneous turbulence. The turbulence intensity of the carrier fluid in a two-phase mixture may be several times that of a single-phase fluid.

A number of investigations have dealt with a theoretical description of the particles–turbulence interaction (Abramovich 1970; Elghobashi & Abou-Arab 1983; Shuen *et al.* 1985; Abou-Arab & Roco 1988; Shraiber *et al.* 1990). They approached the problem by extending the single-phase models of turbulence (mixing length theory, the k–ϵ model etc.) to calculate the particles–turbulence interaction. One of the first attempts was that of Abramovich (1970), who used Prandtl's (1925) mixing-length theory and derived a simple expression determining the dependence of the velocity fluctuations intensity on the carrier fluid and the particles' physical properties as well as the flow regime parameters. This theory does not result in turbulence modulation and, naturally, may be used only for estimation of the fluctuations in two-phase flows with fine particles. In this case, fairly good agreement between the theoretical and experimental data was achieved. Later this approach was used in a number of investigations to study the effects of flow nonequilibrium, gravity etc. on the turbulence intensity (Abramovich *et al.* 1984). Yarin & Hetsroni (1994a,b, this issue, pp.1–25) have used the modified mixing-length theory to calculate the effect of the particle-size distribution on the turbulence of the carrier fluid and to estimate the level of the temperature fluctuations in two-phase monodisperse and polydisperse mixtures.

A number of theoretical methods have been developed to describe turbulent flows laden with coarse particles. The turbulence modulation has been investigated by Al-Taweel & Landau (1977) Parthasarathy & Faeth (1990) and Yuan & Michaelides (1991). In the latter work a simple estimation was proposed by using the turbulent kinetic energy balance. Gore & Crowe (1989) attempted to generalize the experimental data on the particles–turbulence interaction and suggested the use, as a basic parameter, of the ratio of particle size to a characteristic length scale of flow. The analysis of the data for various types of particle-laden flows has shown that there exists a fairly distinct particle dimensionless size below which particles suppress turbulence and above which they enhance it. This boundary corresponds to the ratio particle diameter/turbulence scale ≈ 0.1.

Hetsroni (1989) used the data of Gore & Crowe (1989) and suggested that larger particles, with $Re_p > 110$, cause vortex shedding behind them. These vortices actually cause enhancement of the turbulence, i.e. the particles cause energy to be transferred from the average velocity to higher frequencies.

Previous numerical studies of multiphase flows used turbulence models, such as k–ϵ models, and extrapolated the coefficients from single-phase flows (e.g. Danon *et al.* 1974). These computations may serve useful purposes, but do little to enhance the physical understanding of multiphase flows and will, therefore, not be reviewed here.

The aim of the present study is to evaluate the effect of particle size on the turbulence of the carrier fluid. We combine the modified mixing-length theory and turbulent kinetic energy approach. As a result, we predict the suppression or enhancement of turbulence, depending on the regime parameters of particle-laden turbulent flows.

PHYSICAL MODEL

Consider particle-laden turbulent flow with a velocity gradient, as in figure 1. A fluid element of the admixture moves along the $0y'$ axis to a distance equal to a mixing length. In single-phase flow, such a motion of a fluid element gives the appearance of the average velocity gradient

$$v_0' = l_0 \frac{\partial U}{\partial y'}$$

where l_0 is the mixing length and U is the average velocity. In two-phase flow with a velocity gradient, these fluctuations will depend not only on the velocity gradient of the carrier fluid but also on the mass of the particles inside the fluid element, their size distribution etc. These, in turn, determine the viscous and inertial interactions between the particles and the fluid. The experimental evidence indicates that small particles suppress turbulence, most likely due to inertia effects; while larger particles enhance turbulence, most likely due to vortex shedding (Hetsroni 1989). To describe the physics of turbulence modulation both of these mechanisms will have to be accounted for.

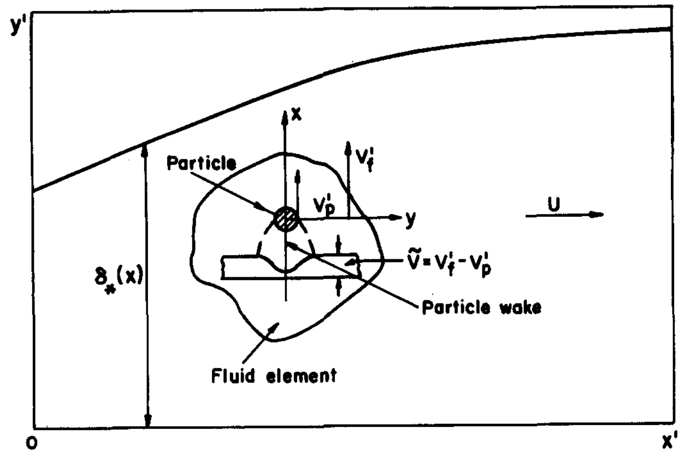

Figure 1. Schematic of particle-laden flow; x' and y' are the longitudinal and transversal coordinates in the laboratory frame of reference, $\delta_*(x)$ is the boundary thickness of the external flow.

MODELING

To describe the fluctuations of the carrier fluid and the particles, we use the momentum balance equations of a fluid element and a single particle, as well as the equations describing the turbulent flow behind coarse particles. The former equations govern the fluctuations due to the motion of the fluid element in a field with velocity gradients, whereas the latter ones govern turbulence modulation in the particle wakes.

Assuming that only inertial and viscous forces act on a particle, the momentum equations for a fluid element (including the particles) and a spherical particle are†

$$\frac{\mathrm{d}}{\mathrm{d}t}(v'_\mathrm{f} + \gamma v'_\mathrm{p}) = 0 \qquad [1]$$

and

$$m_\mathrm{p}\frac{\mathrm{d}v'_\mathrm{p}}{\mathrm{d}t} = \tfrac{1}{2}c_\mathrm{D}\rho_\mathrm{f}\tilde{v}|\tilde{v}|f_\mathrm{p}, \qquad [2]$$

where v'_f and v'_p are the carrier fluid and particle velocity fluctuations due to the turbulent fluid element motion in gradient flow, m_p and f_p are the mass and cross-sectional area of the particle, $\gamma = M_\mathrm{p}/M$ is the mass content of the particles in the fluid element, M and M_p are the mass of the fluid in the fluid element and the mass of the particle respectively, c_D is the drag coefficient of the particle, $\tilde{v} = v'_\mathrm{f} - v'_\mathrm{p}$ is the relative velocity between the carrier fluid and the particle and ρ_f is the fluid density.

The drag coefficient in [2] is given by (Boothroyd 1971):‡

$$c_\mathrm{D} = \frac{24}{\mathrm{Re}}(1 + 0.15\,\mathrm{Re}^{0.687}), \qquad [3]$$

where $\mathrm{Re} = |v|d/\nu$, $\nu = \mu/\rho_\mathrm{f}$, μ is the fluid viscosity and d is the diameter of the particle.

Equations [1] and [2] are to be integrated in the time interval $0 < t < \tau^*$, where the interaction time τ^* is determined according to Abramovich (1970):

$$\tau^* = \frac{l}{|v'_\mathrm{p}|} \qquad [4]$$

(l is the mixing length).

The initial conditions for [1] and [2] are:

$$@\ t = 0, \qquad v'_\mathrm{f} = v'_0, \quad v'_\mathrm{p} = 0. \qquad [5]$$

†The assumptions for [1], [2] and [5] are described in the first part of this series of papers (Yarin & Hetsroni 1994a).
‡Correlation [3] was obtained by Schiller & Nauman (1933).

For the description of the turbulent motion in the wake of a coarse particle, we use the system of equations of the carrier fluid momentum balance and of continuity and the equation for the specific turbulent kinetic energy. In the *boundary layer approximation*, assuming a *quasi-stationary* character of the wake, these equations are

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{y}\frac{\partial}{\partial y}(y\overline{u'v'}), \quad [6]$$

$$\frac{\partial u}{\partial x}+\frac{1}{y}\frac{\partial yv}{\partial y}=0 \quad [7]$$

and

$$u\frac{\partial \frac{q^2}{2}}{\partial x}+v\frac{\partial \frac{q^2}{2}}{\partial y}+\frac{1}{y}\frac{\partial}{\partial y}\left[yv'\left(\frac{p}{\rho}+\frac{q^2}{2}\right)\right]+\overline{u'v'}\frac{\partial u}{\partial y}+15v\overline{\left(\frac{\partial u'}{\partial x}\right)^2}=0. \quad [8]$$

Here we use the frame of reference associated with the particle, u and v are the mean velocity components, corresponding to the x and y axes, respectively (see figure 1), u' and v' are the velocity fluctuations, $q^2/2$ is the specific turbulent kinetic energy and p is the pressure.

The boundary conditions for the system of equations [6]–[8] are:

$$\left.\begin{array}{lll} @\ y=0, & \dfrac{\partial u}{\partial y}=0,\quad v=0,\quad \dfrac{\partial \frac{q^2}{2}}{\partial y}=0 \\ \text{and} & \\ @\ y=\delta, & u=u_\delta,\quad \dfrac{q^2}{2}=0; \end{array}\right\} \quad [9]$$

where δ is the half-width of the wake, $u_\delta=\tilde{v}$.

TURBULENCE MODULATION

Consider the flow behind a coarse particle moving in the fluid element, and assume that the velocity difference in the wake is small. In this case the solution of the hydrodynamical problem for the mean velocity far enough from the particle may be found in the self-similar form (Swain 1929):

$$\frac{u_1}{u_{1m}}=f(\eta),\quad u_{1m}=\mathrm{A}x^{\alpha}, \quad [10]$$

where $u_1=u_\delta-u$ is the velocity defect ($u_1 \ll u_\delta$ at great distance from the particle), $u_{1m}=u_\delta-u_m$ (u_m is the velocity at the wake axis), $f(\eta)$ is some function of the variable $\eta=y/\delta$, $\delta=\mathrm{B}x^{\Delta}$; and A, B and Δ are constants. The function $f(\eta)$ takes the form (Abramovich *et al.* 1984):

$$f(\eta)=(1-\eta^{3/2}),\quad u_{1m}=\mathrm{A}x^{-2/3}, \quad [11]$$

where

$$\mathrm{A}=u_\delta c_{\mathrm{D}}^{1/3}d^{2/3}\left(\frac{0.138^3}{4\cdot\beta^4}\right)^{1/3},\quad \mathrm{B}=c_{\mathrm{D}}^{1/3}d^{2/3}\left(\frac{27\beta^2}{4\cdot 0.129}\right)^{1/3},$$

β is an empirical constant ($\beta=l/\delta$) and d is the particle diameter.

Neglecting the effect of the pressure gradient on the turbulent kinetic energy, we arrive at the integral representation of [8]:

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^\delta \tfrac{1}{2}uq^2y\,\mathrm{d}y+\int_0^\delta \overline{u'v'}\frac{\partial u}{\partial y}y\,\mathrm{d}y=-\int_0^\delta \epsilon y\,\mathrm{d}y, \quad [12]$$

where ϵ is the dissipation, given by Kolmogorov (1942),

$$\epsilon = c\frac{(q^2)^{3/2}}{L}. \qquad [13]$$

Here c is a constant, L is the integral scale of turbulence, which is proportional to the size of the mixing zone and $L = c_1\delta$; c_1 is a constant. The values of the constants c and c_1 in the expressions of the dissipation and the integral scale of turbulence may be taken as $c = (2\sqrt{3})^{-1}$ and $c_1 = 0.2$. (Townsend 1956).

Assume that the turbulent fluctuation energy distribution in the cross sections of the wake also has a self-similar form:

$$\frac{q^2}{q_x^2} = \varphi(\eta). \qquad [14]$$

Here q_x^2 is the turbulence fluctuation energy at the wake axis and $\varphi(\eta)$ is a function of the variable η.

By using [14] and the expression $\overline{u'v'} = l^2|\partial u/\partial y|(\partial u/\partial y)$, we rearrange [12] by substituting [11], [13] and [14]. Then we obtain

$$\tfrac{1}{2}u_\delta\frac{d}{dx}(q_x^2\delta^2)\cdot I_1 + \frac{c}{c_1}(q_x^2)^{3/2}\delta I_2 - \beta^2 u_{1m}^2\delta I_3 = 0, \qquad [15]$$

where

$$I_1 = \int_0^1 \varphi(\eta)\eta\,d\eta \qquad [16]$$

$$I_2 = \int_0^1 \varphi(\eta)^{3/2}\eta\,d\eta \qquad [17]$$

and

$$I_3 = \int_0^1 [f'(\eta)]^2|f'(\eta)|\eta\,d\eta. \qquad [18]$$

To evaluate the integrals I_1 and I_2, we use the experimental data of Uberoi & Freymouth (1970) on the turbulent kinetic energy distribution across the wake of a sphere. As a result we obtain $I_1 = 0.132$ and $I_2 = 0.112$. To evaluate the integral I_3, we use [11] and obtain $I_3 = 0.599$. By using the experimental data on the mean velocity distribution along the wake axis we find the value of the constant $\beta = 0.453$. This value is larger than that of a turbulent axisymmetric jet; a similar result was also observed for planar wakes (Abramovich *et al.* 1984).

Assume that the turbulent kinetic energy distribution along the wake axis has the form

$$q_x^2 = Nx^\omega, \qquad [19]$$

where N and ω are constants. Substituting expression [19] in [15], and requiring that the result be independent of x, we arrive at $\omega = -4/3$. We also obtain an algebraic equation for the constant N:

$$e_1 N^{3/2} + e_2 N + e_3 = 0, \qquad [20]$$

where

$$e_1 = \frac{c}{c_1}I_2, \quad e_2 = -\tfrac{5}{6}u_\delta B I_1, \quad e_3 = -\beta^2 A^3 I_3.$$

Introducing the new variable $Z = n + (e_2/3e_1)$, $(n = N^{1/2})$, we rearrange [20] in the form

$$Z^3 + 3pZ + 2g = 0, \qquad [21]$$

where

$$2g = \frac{2e_2^3}{27e_1^3} + \frac{e_3}{e_1}, \quad 3p = -\frac{e_2^2}{3e_1^2}.$$

The third order polynomial in [21] can be solved, and the solution depends on the sign of $D = g^2 + p^3$. The latter may be written in the form

$$D = \frac{e_3}{e_1}\left(\frac{e_2^3}{27e_1^3} + 0.25\frac{e_3}{e_1}\right). \qquad [22]$$

As the ratios $e_3/e_1 < 0$ and $e_2^3/e_1^3 < 0$, the value of $D > 0$. Therefore, [21] has one real and two imaginary roots. The real root is

$$Z = Z' + Z'', \qquad [23]$$

where

$$Z' = \sqrt[3]{-g + \sqrt{g^3 + p^3}}; \quad Z'' = \sqrt[3]{-g - \sqrt{g^3 + p^3}}.$$

By using the expressions for the coefficients e_1, e_2 and e_3, we obtain

$$Z' = u_\delta c_D^{1/3} d^{2/3} \sqrt[3]{M_1 + \sqrt{M_1^2 + M_2^3}} \qquad [24]$$

and

$$Z'' = u_\delta c_D^{1/3} d^{2/3} \sqrt[3]{M_1 - \sqrt{M_1^2 + M_2^3}}, \qquad [25]$$

where

$$M_1 = \left(\frac{c_1}{c}\right)\beta^2\left[\frac{125}{216}\frac{1}{4\cdot 0.129}\left(\frac{I_1}{I_2}\right)^3\left(\frac{c_1}{c}\right)^2 + 0.5\frac{0.138^3}{4\cdot\beta^4}\left(\frac{I_3}{I_2}\right)\right]$$

and

$$M_2 = -\frac{1}{9}\frac{25}{36}\left(\frac{27\cdot\beta^2}{4\cdot 0.129}\right)^{2/3}\left(\frac{I_1}{I_2}\right)\left(\frac{c_1}{c}\right)^2.$$

The real root of [21] is

$$Z = u_\delta c_D^{1/3} d^{2/3} M_3, \qquad [26]$$

where $M_3 = \sqrt[3]{M_1 + \sqrt{M_1^2 + M_2^3}} + \sqrt[3]{M_1 - \sqrt{M_1^2 + M_2^3}}$. By using [26] we find the value of the constant N:

$$N = u_\delta^2 c_D^{2/3} d^{4/3} M_5, \qquad [27]$$

where

$$M_5 = (M_3 + M_4)^2; \quad M_4 = \frac{5}{3\cdot 6}\left(\frac{c_1}{c}\right)\left(\frac{I_1}{I_2}\right)\left(\frac{27\cdot\beta^2}{4\cdot 0.129}\right)^{1/3}.$$

Note that the coefficients M_1, M_2, M_3, M_4 and M_5 depend only on the parameter β.

Now we obtain the expression for the turbulent energy on the axis of the wake:

$$q_x^2 = \tilde{v}^2 c_D^{2/3} d^{4/3} M_5 x^{-4/3}. \qquad [28]$$

It is seen that q_x^2 depends on the particle size and its relative velocity.

Equations [10], [22], [14] and [28] enable one to determine the local characteristics of the flow in the wake of a single particle. To estimate the effect of turbulence modulation on the fluctuations intensity in particle-laden flow we need to account for an ensemble of particles. To do this, we first calculate the overall fluctuation energy in the wake of a single particle, integrating the distribution of the energy of the fluctuations:

$$q_t^2 = 2\pi \int_0^{l_w}\left(\int_0^{\delta} q^2 y \, dy\right) dx. \qquad [29]$$

Here q_t^2 is the overall energy of the fluctuations in the wake of a single particle and l_w is the length of the wake.

Note, that the integral in [29] is converging on account of the finite wake length. Substitution of [14] and [19] in [29] yields

$$q_t^2 = 6\pi I_1 N B^2 l_w^{1/3}. \qquad [30]$$

The energy of the fluctuations generated by the particles in a unit volume of two-phase mixture is

$$q_v^2 = \frac{q_t^2 n}{L_f^3}, \qquad [31]$$

where n is the number of particles in the volume L_f^3. Using the equation of mass balance, one obtains

$$L_f^3 = \frac{n\pi d^3 \rho_{pf}}{6\gamma}, \qquad [32]$$

where $\rho_{pf} = \rho_p/\rho_f$ is the phase density ratio. Substitution of [11], [27], [30] and [32] in [31] yields the expression for the total energy of the fluctuations:

$$q_v^2 = \frac{36}{\rho_{pf}} I_1 u_\delta^2 c_D^{4/3} \gamma M_5 \left(\frac{27 \cdot \beta^2}{4 \cdot 0.129}\right)^{2/3} \left(\frac{l_w}{d}\right)^{1/3}. \qquad [33]$$

Since it was assumed that the particle is inside the fluid element during its life time one may assume that the maximum particle wake length is of the order of the diameter of the fluix element, i.e. $l_w \propto L_f$. This, with [32], yields the following value of the ratio $(l_w/d)^{1/3}$ for dilute ($n = 1$) two-phase flows:

$$\left(\frac{l_w}{d}\right)^{1/3} = \Omega\left(\frac{\rho_{pf}}{\gamma}\right)^{1/9}, \qquad [34]$$

where Ω is an empirical constant of the order of unity.

Dividing the left- and right-hand sides of [33] by $v_0'^2$ and substituting for the latter expression as well as the values of the integral I_1, $u_\delta = \tilde{v}$ and constants β, c, c_1, we get:

$$\bar{q}_v^2 = 52.55\ \Omega \bar{\tilde{v}}^2 \left(\frac{\gamma}{\rho_{pf}} c_D^{3/2}\right)^{8/9}, \qquad [35]$$

where $\bar{q}_v^2 = q_v^2/v_0'^2$ and $\bar{\tilde{v}} = \tilde{v}/v_0' = [1 - \bar{v}_p'(1 + \gamma)]$.

When the fluctuations intensity is determined by the turbulence modulation (the effect of the course particles) the turbulence intensity of the carrier fluid is

$$\frac{\sqrt{u'^2}}{\tilde{v}} = \text{const}\left(\frac{\gamma}{\rho_{pf}} c_D^{3/2}\right)^{4/9}. \qquad [36]$$

Namely, in a flow laden with coarse particles, the logarithm of the turbulence intensity $\log(\sqrt{u'^2}/\tilde{v})$ is a linear function of the ratio $(\gamma c_D^{3/2})/\rho_{pf}$, with the coefficient of proportionality equal to 4/9. One may also refer to this result as the 4/9-power law.

In particle-laden flows with extremely coarse particles and high turbulence intensity, the drag coefficient is a very weak function of the relative velocity ($Re_p > 10^3$). In this case the fluctuations energy depends only on the total mass content of the admixture and the phase density ratio (in the form of the 4/9-power law).†

DIMENSIONAL ANALYSIS

A better insight into the nature of the particles–turbulence interaction may be achieved by analyzing the problem in terms of nondimensional parameters. We use as the scales of time, velocity, fluctuations energy and density the following quantities: τ_0 ($\tau_0 = l/v_0'$), l, v_0' and ρ_f. By using the nondimensional variables $\bar{\tau} = t/\tau_0$, $\bar{d} = d/l$, $\bar{v}_f' = v_f'/v_0'$, $\bar{v}_p' = v_p'/v_0'$, $\bar{q}^2 = q^2/v_0'^2$ and $\rho_{pf} = \rho_p/\rho_f$, the equations which describe the system are:

$$\frac{d\bar{v}_p'}{d\bar{\tau}} = \tfrac{3}{4} c_D (\bar{d} \cdot \rho_{pf})^{-1} [1 - \bar{v}_p'(1 + \gamma)]|1 - \bar{v}_p'(1 + \gamma)|, \qquad [37]$$

$$c_D = \frac{24}{Re_p}[1 + 0.15 \cdot Re_p^{0.687}], \qquad [38]$$

†This effect may be essential when gravity forces are of the same order as or even higher than the aerodynamic drag. For example, $Re_p = 780$ for free-falling particles $d = 2$ mm (Mizukami *et al.* 1992); see the later section "Effect of gravity".

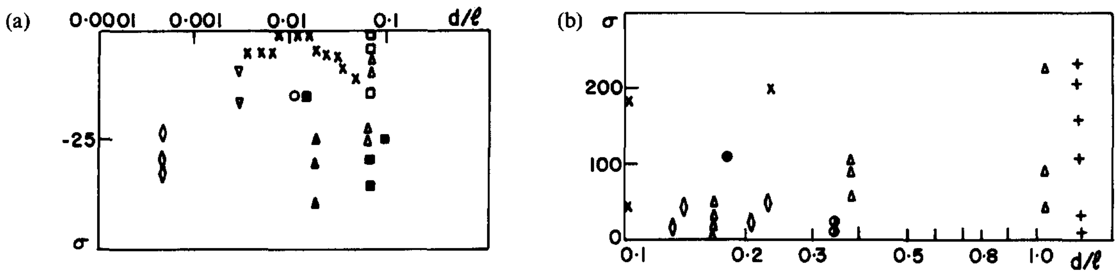

Figure 2a. Dependence of the enhancement coefficient on the particle size/turbulence scale ratio [enlarged from Gore & Crowe (1989)]. Experimental data by: □, Levy & Lockwood (1981); ◇, Hetsroni & Sokolov (1971); △, Tsuji *et al.* (1984); ▽, Modarress *et al.* (1984a); ○, Modarress *et al.* (1984b); ×, Shuen *et al.* (1985); ■, Zisselmar & Molerus (1979); ▲, Maeda *et al.* (1980).

Figure 2b. Dependence of the enhancement coefficient on the particle size/turbulence scale ratio [enlarged from Gore & Crowe (1989)]. Experimental data: +, Tsuji & Morikawa (1982); △, Tsuji *et al.* (1984); ×, Shuen *et al.* (1985); ●, Lee & Durst (1982); ◇, Sun (1986); ◑, Parthasarathy & Faeth (1987).

$$\mathrm{Re}_p = \mathrm{Re}_{p0}[1 - \bar{v}'_p(1+\gamma)]; \quad \mathrm{Re}_{p0} = \frac{d v'_0}{\nu}, \qquad [39]$$

$$\bar{q}^2_v = 52.55\Omega\left(\frac{\gamma}{\rho_{pf}} c_D^{3/2}\right)^{8/9} [1 - \bar{v}'_p(1+\gamma)]^2 \qquad [40]$$

and

$$\bar{q}^2_\Sigma = \bar{q}^2_f + \bar{q}^2_v, \qquad [41]$$

where $\bar{q}^2_\Sigma$ is the total turbulent energy and $\bar{q}^2_f = 3\bar{v}'^2_f$ and $\bar{q}^2_v$ are the turbulent kinetic energy due to the velocity gradient and the turbulence modulation in the wakes behind the particles, respectively.

It is seen that [37]–[41] contain four nondimensional parameters: $\bar{d}$, Re_{p0}, γ and ρ_{pf}. Therefore, the experimental data for the particles–turbulence interaction may be described using these four criteria of similarity:

$$\bar{d} = idem, \quad \mathrm{Re}_{p0} = idem, \quad \gamma = idem, \quad \rho_{pf} = idem. \qquad [42]$$

It is clear that an attempt to describe the particles–turbulence interaction in terms of less than four nondimensional variables would be unsuccessful. This is evident in figures 2a and 2b, where the data of a number of typical particle-laden flows are shown [taken from Gore & Crowe (1989)†]. The ratio diameter/turbulence scale does not generalize the experimental data for different particles sizes, and cannot be used as the sole criterion of similarity. It may be used only to estimate the domains of decreasing and increasing turbulence intensity.

In particular cases when the value of the criterion $\bar{d}$ is significantly smaller or larger than unity (fine or coarse particles), the analysis of the problem may be simplified drastically.

The first particular case corresponds to two-phase flows with relatively small Re_p (small relative velocities), <400. Naturally, in this case the turbulence modulation by wake shedding mechanism does not take place and the process of the particles–turbulence interaction is determined only by one parameter: the total mass content. This result reflects, as a matter of fact, the leading role of particle inertia in particle-laden flows with fine particles. In the second particular case, of coarse particles, their absolute velocities are relatively small and the level of fluctuations is determined by the process of turbulence modulation due to wake shedding. Correspondingly, the turbulent kinetic energy depends only on the ratio $(\gamma/\rho_{pf})c_D^{3/2}$. For the very large particles (with $\mathrm{Re}_p \gtrsim 10^3$), the drag coefficient is approximately constant. In this case, the level of turbulent fluctuations is determined by the ratio of the total mass content of the admixture to the phase density ratio (γ/ρ_{pf}).

These results are illustrated in figures 3–6, where the enhanced turbulence intensity is plotted vs γ and γ/ρ_{pf} for particle-laden turbulent jets and pipe flows.

†The particles–turbulence interaction may be characterized by the ratio of the turbulence change due to particles in a particle-laden flow to the turbulence intensity in a corresponding single-phase flow: $\sigma = (\sqrt{\overline{u_p'^2}} - \sqrt{\overline{u'^2}})/\sqrt{\overline{u'^2}} \cdot 100$, where $\sqrt{\overline{u_p'^2}}$ and $\sqrt{\overline{u'^2}}$ are the turbulence intensities in particle-laden and pure flow, respectively (Gore & Crowe 1989). This coefficient may be called the "enhancement coefficient".

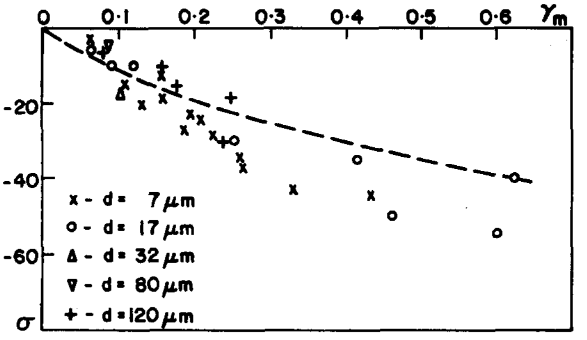

Figure 3. Dependence of the enhancement coefficient on the total mass content of the admixture in the turbulent jet. Data by Laats & Frishman (1972); ---, $\sigma = -\gamma(1+\gamma)^{-1}$, γ_m is the mass content of the admixture at the jet axis.

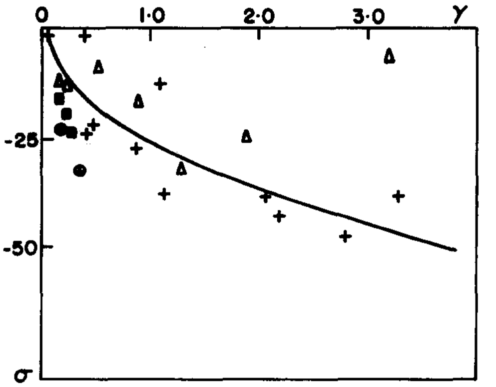

Figure 4. Dependence of the enhancement coefficient on the total mass content of the admixture. Experimental data by: +, Tsuji & Morikawa (1982); △, Tsuji *et al.* (1984); ■, Zisselmar & Molerus (1979); ⊕, Maeda *et al.* (1980).

In figure 3, experimental data for the carrier fluid fluctuations at the axis of an axisymmetric jet (Laats & Frishman 1973) is presented. The particles sizes in these experiments changed within the limits of 7–120 μm and the total mass content within $0 < \gamma_m < 0.6$. The graph also plots the curve $\sigma = -\gamma(1+\gamma)^{-1}$, corresponding to the asymptotic case for the turbulence intensity at the limiting case $\bar{d} \to 0$. As seen in figure 3, in the plane σ–γ all the points corresponding to various particle sizes are grouped about a single curve $\sigma(\gamma_m)$. A similar behavior is seen in figure 4, where the experimental data for pipe flows is plotted in the form of the turbulence enhancement coefficient vs the total mass content of the admixture (Tsuji & Morikawa 1982; Tsuij *et al.* 1984; Zisselmar & Molerus 1979; Maeda *et al.* 1980). It is also seen that, in all cases, an increase in the total mass content of the fine particles leads to a decrease in the level of fluctuations.

The effect of the mass content on the enhancement coefficient is depicted in figure 5 for various locations in the pipe. It is seen that, in spite of the large scattering of the experimental data, the above-mentioned tendency is observed at each y/R position in the central part of the flow where the effect of the pipe walls is smaller.

In figure 6 the data from experiments on air with coarse particles suspended in it, flowing in horizontal and vertical pipes (Tsuji & Morikawa 1982; Tsuji *et al.* 1984) is plotted, in nondimensional form. This representation supports the conclusion that there is a dependence of σ on the single parameter γ/ρ_{pf} for coarse particles of different sizes. In figure 6 it is also seen that there is

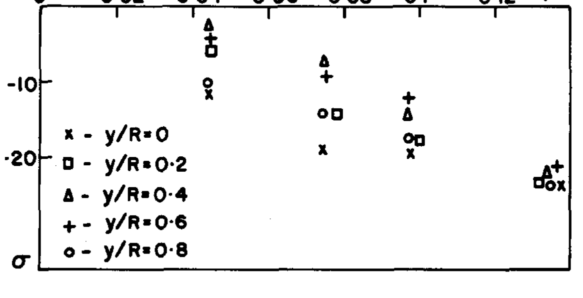

Figure 5. Dependence of the enhancement coefficient on the total mass content at various cross sections in the pipe flow. Experimental data by Zisselmar & Molerus (1979).

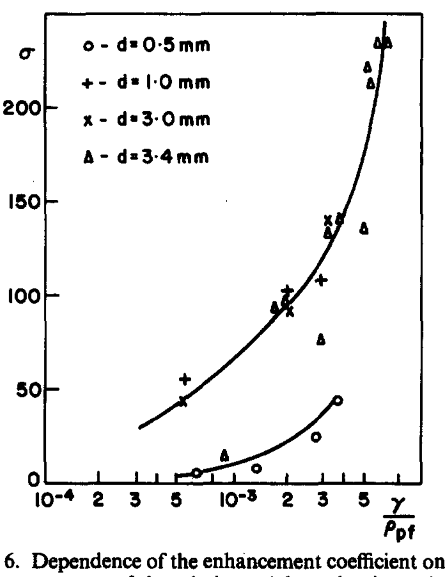

Figure 6. Dependence of the enhancement coefficient on the total mass content of the admixture/phase density ratio for the pipe axis. Experimental data by: △, Tsuji & Morikawa (1982); ×, +, ○, Tsuji *et al.* (1984).

an essential distinction between the curves corresponding to particle-laden flows with fine ($d = 0.5$ mm) and coarse ($d = 1$ to 3 mm) particles. This results from the fact that the dependence of the enhancement coefficient on the ratio $\gamma/\rho_{\mathrm{pf}}$ has been proven only for sufficiently large particles. In a mixture with particles of smaller sizes, the effect of the particle diameter on the drag coefficient is important. Due to this effect, the curve $\sigma(\gamma/\rho_{\mathrm{pf}})$ for the smallest particles is separated in figure 6 from the others.

EFFECT OF THE PARTICLE-SIZE DISTRIBUTION

The velocity fluctuation in a polydisperse mixture is now calculated. In the case of a bidisperse admixture (a carrier fluid with particles of two sizes), one needs to solve a system of nonlinear equations which describe the carrier fluid and particle velocity fluctuations and the corresponding relative velocities $\tilde{v}_1 = v_{\mathrm{f}}' - v_1'$ and $\tilde{v}_2 = v_{\mathrm{f}}' - v_2'$ (subscripts 1 and 2 relate to the two groups of particles):

$$\bar{v}_{\mathrm{f}}' + \gamma_1 \bar{v}_1' + \gamma_2 \bar{v}_2' = 1, \qquad [43]$$

$$\frac{\mathrm{d}\bar{v}_1'}{\mathrm{d}\bar{\tau}} = \tfrac{3}{4} c_{\mathrm{D}}^{(1)} (\bar{d}_1 \rho_{\mathrm{pf}})^{-1} [1 - \bar{v}_1'(1+\gamma_1) - \gamma_2 \bar{v}_2'] |1 - \bar{v}_1'(1+\gamma_1) - \gamma_2 \bar{v}_2'|, \qquad [44]$$

$$\frac{\mathrm{d}\bar{v}_2'}{\mathrm{d}\bar{\tau}} = \tfrac{3}{4} c_{\mathrm{D}}^{(2)} (\bar{d}_1 \rho_{\mathrm{pf}})^{-1} [1 - \gamma_1 \bar{v}_1' - (1+\gamma_2 \bar{v}_2')] |1 - \gamma_1 \bar{v}_1' - (1+\gamma_2) \bar{v}_2'|, \qquad [45]$$

$$c_{\mathrm{D}}^{(i)} = \frac{24}{\mathrm{Re}_{\mathrm{p}}^{(i)}} [1 + 0.15\, \mathrm{Re}_{\mathrm{p}}^{(i)0.687}], \quad i = 1, 2, \qquad [46]$$

$$\mathrm{Re}_{\mathrm{p}}^{(1)} = \mathrm{Re}_{\mathrm{p0}}^{(1)} [1 - \bar{v}_1'(1+\gamma_1) - \gamma_2 \bar{v}_2']; \quad \mathrm{Re}_{\mathrm{p0}}^{(1)} = \frac{d_1 v_0'}{\nu} \qquad [47]$$

and

$$\mathrm{Re}_{\mathrm{p}}^{(2)} = \mathrm{Re}_{\mathrm{p0}}^{(2)} [1 - \gamma_1 \bar{v}_1' - (1+\gamma_2) \bar{v}_2']; \quad \mathrm{Re}_{\mathrm{p0}}^{(2)} = \frac{d_2 v_0'}{\nu}, \qquad [48]$$

where $v_1' = v_{\mathrm{p}}'^{(1)}$; $\bar{v}_2' = \bar{v}_{\mathrm{p}}'^{(2)}$, the indices 1 and 2 correspond to the larger and smaller particle sizes, respectively.

It is emphasized that if the Stokes formula $c_{\mathrm{D}}^{(i)} = 24\, \mathrm{Re}_{\mathrm{p}}^{(i)}$ is applicable, the system of equations [43]–[48] has an analytical solution (Yarin & Hetsroni 1994a).

The average energy of the fluctuations per unit volume of a polydisperse mixture with particles of different sizes is now calculated. For the particles of some fraction with diameter d_i, we can write the following correlation:

$$q_{\mathrm{t}}^2 = 6\pi \mathrm{I}_1 \mathrm{N}_i \mathrm{B}_i^2 l_{\mathrm{w}i}^{1/3}. \qquad [49]$$

Assume that the polydisperse mixture contains k_i particles of diameter d_i. Therefore,

$$q_{\mathrm{v}}^2 = 36 \mathrm{I}_1 \frac{\gamma}{\rho_{\mathrm{pf}}} \left(\frac{27 \cdot \beta^2}{4 \cdot 0.129} \right)^{2/3} \Omega M_5 \frac{\sum_{i=1}^{n} k_i \tilde{v}_i^2 c_{\mathrm{D}i}^{4/3} \left(\frac{\rho_{\mathrm{pf}}}{\gamma_i} \right)^{1/9} d_i^3}{\sum_{i=1}^{n} k_i d_i^3}. \qquad [50]$$

Noting that

$$\frac{k_i d_i^3}{\sum_{i=1}^{n} k_i d_i^3} = \frac{\gamma_i}{\gamma}$$

and substituting in [50], we find

$$q_{\mathrm{v}}^2 = 52.55\, \Omega \sum_{i=1}^{n} \tilde{v}_i^2 \left(\frac{\rho_{\mathrm{pf}}}{\gamma_i} c_{\mathrm{D}i}^{3/2} \right)^{8/9}. \qquad [51]$$

Applying the general equations [43]–[50] to a bidisperse mixture, we see that the turbulence modulation in it depends on the particle Reynolds numbers $\mathrm{Re}_{\mathrm{p0}}^{(1)}$ and $\mathrm{Re}_{\mathrm{p0}}^{(2)}$, their sizes $\bar{d}_1$ and $\bar{d}_2$, the mass content of the fine and coarse particles γ_1 and γ_2, and the phase density ratio.

EFFECT OF GRAVITY

In the previous section we considered the modulation of turbulence as affected by inertial and aerodynamic forces, without considering gravity. We now consider an admixture, with coarse particles, in a gravitational field. Actually, the gravity forces may be of the same order as or even higher than the aerodynamic drag.

Girshovich & Leonov (1979) shows that the effect of gravity on the turbulent structure of particle-laden jets is significant. It manifests itself in changes in the fluctuations intensity, turbulent shear stress etc. The effect of gravity depends on the ratio of the initial velocity fluctuations to the terminal settling velocity and particle sizes, their total mass content etc.

Here we limit our attention to the effect of gravity on tubulence modulation in vertical particle-laden flows. Other effects, like the variation in particle concentration in horizontal flows, are not considered.

Assume that the gravity force is in equilibrium with the aerodynamic drag force:

$$\tfrac{1}{2} c_D \tilde{v}^2 \rho_f f_p = mg, \qquad [52]$$

where g is the acceleration due to gravity, $\tilde{v}$ is the particle relative velocity: ultimate velocity (soaring velocity). From [52] we obtain

$$\tilde{v}^2 = \frac{4}{3}\frac{\rho_{pf}}{c_D} dg. \qquad [53]$$

We see that in the given case the relative velocity is determined by the phase density ratio, the drag coefficient, the particle size and the acceleration due to gravity.

Substituting [53] in the expression for fluctuations energy [35] we arrive at

$$q_v^2 = 70.07\,\Omega \frac{\rho_{pf}}{c_D} d \cdot g\left(\frac{\gamma}{\rho_{pf}} c_D^{3/2}\right)^{8/9}. \qquad [54]$$

It is seen that the energy of the carrier fluid velocity fluctuations is proportional to the square root of the particle diameter. For a light admixture, it was found previously that v' depends only on γ when gravity is neglected, which is natural since the total mass content determines only the inertial properties of the admixture.

RESULTS OF THE CALCULATIONS

Consider now the results of the theoretical study of the fluctuations intensity in two-phase flows, when both turbulence generation due to the velocity gradient and turbulence modulation via wake shedding are taking place. First, we show the effect of the ratio of the particle sizes on the characteristics of particle-laden flows. In figure 7, the effect of the parameter $\bar{d}$ on the carrier fluid and the particle fluctuations is shown (the fluctuations are due to the velocity gradient v' and the

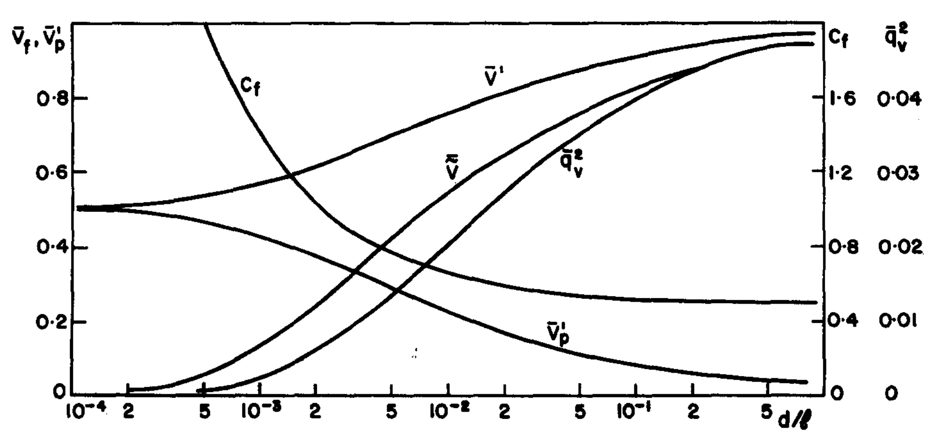

Figure 7. Dependence of the particle and carrier fluid velocity fluctuations on the particle size/turbulence scale ratio; $\gamma = 1$, $Re_{p0} = 500$.

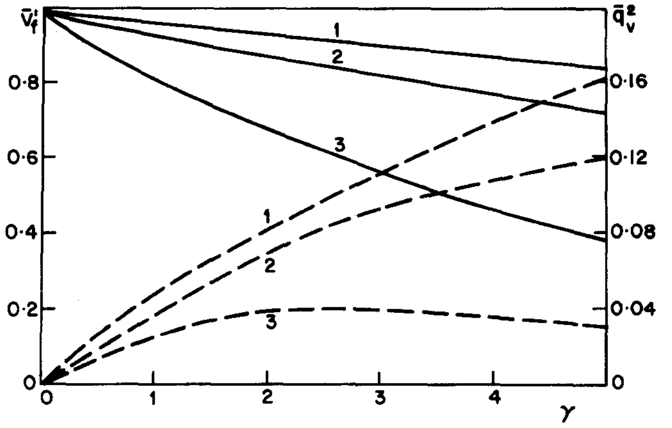

Figure 8. Dependence of the carrier fluid velocity fluctuations on the total mass content of the admixture; $\rho_{pf} = 10^3$, $Re_{p0} = 500$. **1**, $d/l = 0.8$; **2**, $d/l = 0.2$, **3**, $d/l = 0.02$.

relative velocity is proportional to $\sqrt{q_v^2}$). The dependence of the energy of the fluctuations $\bar{q}_v^2$ due to turbulence modulation in the particle wake is also presented in figure 7. It is seen that an increase in the ratio d/l is accompanied by an increase in the carrier fluid fluctuations due to the velocity gradient, and a decrease in the particle fluctuations intensity. The latter causes an increase in the relative velocity and a decrease in the value of the drag coefficient. The energy of the fluctuations due to turbulence modulation in the particle wakes increases monotonously as the particle size/turbulent scale ratio increases.

In figure 8 the carrier fluid fluctuations are plotted as a function of the total mass content of the admixture. For a large d/l ratio, an increase in the parameter γ leads to a decrease in the level of the fluctuations due to the velocity gradient and to a growth in the energy of the fluctuations due to turbulence in the particle wakes. For small values of the d/l ratio, the character of the dependence $\bar{q}_v^2(\gamma)$ is changed. In this case an increase in the total mass content of the admixture leads to an increase in the energy of the fluctuations for relatively small γ. For large γ, an increase in the total mass content of the admixture leads to a decrease in $\bar{q}_v^2$. This effect is due to a decrease in the relative velocity at large values of the total mass content of the admixture.

The effect of the phase density ratio on the carrier fluid turbulence is demonstrated in figure 9. It is seen that an increase in the parameter ρ_{pf} leads to a weak increase in the fluctuations due to the velocity gradient and a decrease in the energy of the fluctuations due to turbulence modulation in the particle wakes.

The effect of Re_{p0} on the carrier fluid fluctuations and the turbulent kinetic energy is illustrated in figure 10. It is seen that an increase in Re_{p0} is accompanied by a decrease in the energy of the

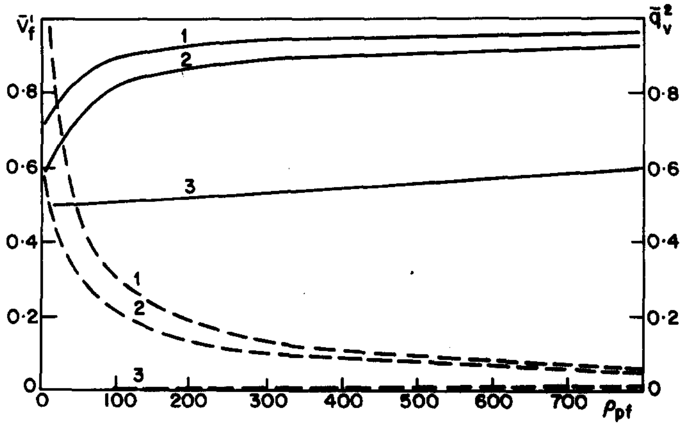

Figure 9. Dependence of the carrier fluid velocity fluctuations on the phase density ratio; $\gamma = 1$, $Re_{p0} = 500$. **1**, $d/l = 0.8$; **2**, $d/l = 0.2$; **3**, $d/l = 0.02$; ——, $\bar{v}_f'$; - - -, $\bar{q}_v^2$.

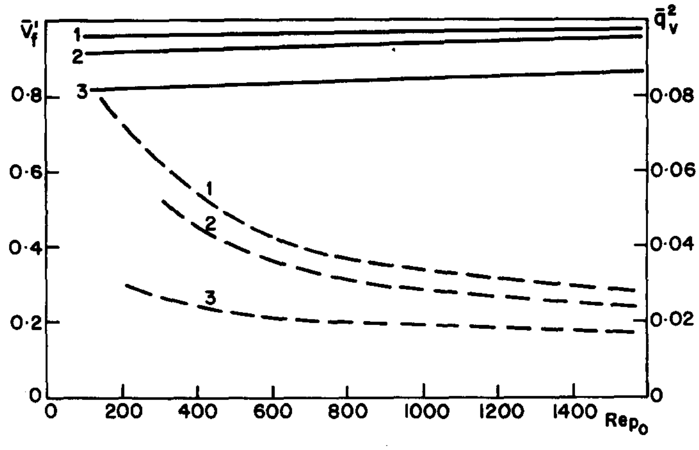

Figure 10. Dependence of the carrier fluid velocity fluctuations on the particle Reynolds number; $\rho_{pf} = 10^3$, $\gamma = 1$. 1, $d/l = 0.8$; 2, $d/l = 0.2$; 3, $d/l = 0.02$; ——, $\bar{v}'_f$; – – –, $\bar{q}^2_v$.

fluctuations due to the turbulence moduluation and a weak growth in the fluctuations due to the velocity gradient.

Note, that for a wide range of the particle/carrier fluid density ratio ($\rho_{pf} > 100$), the absolute values of the energy of the fluctuations are related to the flow around the particles. They are significantly smaller than the values of the energy of the fluctuations due to the velocity gradient. Probably, the modulation of turbulent energy in two-phase flows with coarse heavy particles is affected not only by the fluctuational motion of the particles and carrier fluid due to the velocity gradient, but also by gravity. The latter fact leads to a significant increase in the relative velocity and a corresponding increase in the energy of the fluctuations. These conclusions are used below when comparing the results of the calculations with the experimental data on turbulent fluctuations in particle-laden flows in vertical pipes.

COMPARISON WITH EXPERIMENTS

(a) Homogeneous Turbulence

Homogeneous turbulence is, in a way, ideal for comparison of theoretical models of fluctuations generated by coarse particles with experimental data. The absence of a velocity gradient makes it possible to study the process of turbulence modulation in "pure form".

Some experimental data can be found in Parthasarathy & Faeth (1984) and Mizakami *et al.* (1992). In these studies, the flow generated by the uniform free fall of spherical particles in a space filled by water or air was studied. The range of parameters in these experiments was: $0.5 \leqslant d < 2$ mm; $2.45 \leqslant \rho_{pf} \leqslant 2094$; $0.47 \leqslant c_f \leqslant 2.08$; and $38 \leqslant Re_p \leqslant 780$. The intensity of the fluctuations changed in the range $3.3 < \sqrt{u'^2} < 6.1$ mm/s and $8.6 < \sqrt{u'^2} < 29$ mm/s for water and air, respectively.

In figure 11 the theoretical results of the present study are compared with the experimental data of Parthasarathy & Faeth (1984) and Mizukami *et al.* (1992). Expression [53] is used to calculate the relative velocity in the flow generated by the uniform free fall of spherical particles. The velocity fluctuations of the carrier fluid are plotted vs the nondimensional ratio $(\gamma/\rho_{pf})c_D^{3/2}$. In the graph the theoretical curve corresponding to the 4/9-power law is shown. It is seen that the theoretical result agrees fairly well with the experimental data in the wide range of variation of the flow parameters.

(b) Turbulent Jet

In turbulent particle-laden jets with a coarse admixture the level of fluctuations is determined by the turbulence modulation in the particle wakes and velocity gradients. These gradients may be estimated as the ratio between the velocity at the jet axis u_m and the jet thickness δ. In an axisymmetric jet $u_m \sim x^{-1}$, $\delta \sim x$ and, hence, the velocity gradient is proportional to x^{-2} and sharply decreases as the distance from the nozzle increases. Therefore, one may expect that in the

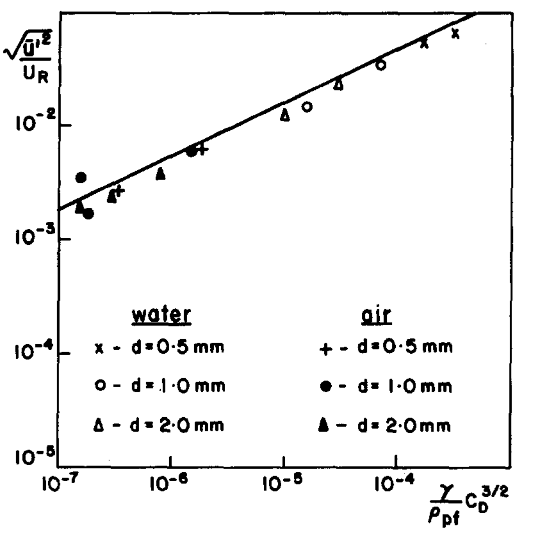

Figure 11. Dependence of the carrier fluid velocity fluctuations on the complex $(\gamma/\rho_{pf})c_D^{3/2}$ (homogeneous turbulence; u_R is the relative velocity). Experimental data by Parthasarathy & Faeth (1990) and Mizukami *et al.* (1992); ——, the 4/9-power law.

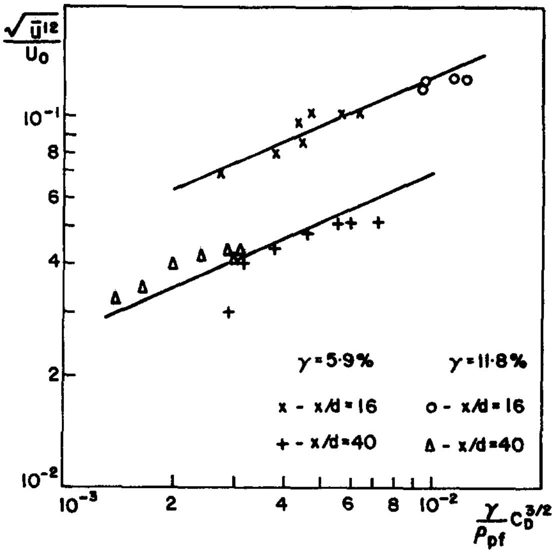

Figure 12. Dependence of the carrier fluid velocity fluctuations on the complex $(\gamma/\rho_{pf})c_D^{3/2}$ (turbulent jet; u_0 is the initial average velocity). Experimental data by Parthasarathy & Faeth (1987); ——, the 4/9-power/law.

far field of the jet the process of turbulence modulation in the particle wakes is the dominant mechanism. In this case the 4/9-power law holds.

In figure 12 the results of the present theory are compared with experimental data on the turbulence intensity in submerged turbulent water jets in still water (Parthasathy & Faeth 1984). In these experiments glass particles of diameter $d = 0.5$ mm and density $\rho_p = 2450$ kg/m^3 were used. The initial loading ratio varied within the range 5.9–11.8%; whereas the initial velocity varied within the range 1.66–1.72 m/s.

The results of the measurements of the turbulent fluctuations in the cross section of the jet are replotted in figure 12 as a function of the nondimensional parameter $(\gamma/\rho_{pf})c_D^{3/2}$. The theoretical curve corresponding to the 4/9-power law is also shown. It is seen that in this case, as previously, the agreement of the theory with the experimental data is fairly good. The difference in the curves $\sqrt{\overline{u'^2}}/u_0$ on $[(\gamma/\rho_{pf})c_D^{3/2}]$, corresponding to different cross sections of the jet, is related to the difference in the particle and carrier fluid relative velocities at different distances from the nozzle. The absence of data on $\bar{v}$ does not allow us to replot the experimental results in the form of the sole dependence of $\sqrt{\overline{u'^2}}/u_0$ on $[(\gamma/\rho_{pf})c_D^{3/2}]$ in different cross sections of the jet.

(c) Pipe Flow

Particle-laden pipe flow has a number of characteristic features due to the effect of the velocity gradient and the particle wakes turbulence interplay. This interplay manifests itself in an essentially nonuniform distribution of velocity fluctuations in pipe cross sections, emergence of the regions of enhancement and suppression of turbulence. As a result there should be a discrepancy between the experimental data and the 4/9-power law.

There are some experimental studies on the velocity fluctuations in turbulent pipe flows with coarse particles. To compare the theoretical predications with experimental data we chose the data of Tsuji *et al.* (1984). In the latter, detailed data on the $\sqrt{\overline{u'^2}}$ fluctuations distribution for various values of the total mass content is presented. The data of Tsuji *et al.* (1984) has been used also because the $\sqrt{\overline{u'^2}}$ profiles are axisymmetric in vertical flow.

In figure 13 the experimental data by Tsuji *et al.* (1984) are replotted in the form of the dependence of the enhancement coefficient σ on the nondimensional radius. It is seen that in the region with a small velocity gradient (near the pipe axis) there is a noticeable increase in the fluctuations. This is a result of turbulence modulation in the wakes of the particles. Near the wall, the enhancement coefficient decreases. In particle-laden flow with a fine admixture ($d \lesssim 1$ mm) there

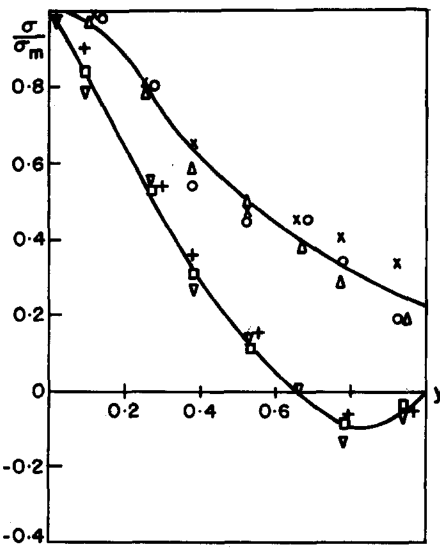

Figure 13. Distribution of the enhancement coefficient in the cross section of the pipe, σ_m corresponds to the pipe axis. Experimental data by Tsuji *et al.* (1984). $d = 3$ mm: ×, $\gamma/\rho_{pf} = 5.88 \cdot 10^{-4}$; ○, $\gamma/\rho_{pf} = 2.2 \cdot 10^{-3}$; △, $\gamma/\rho_{pf} = 3.3 \cdot 10^{-3}$. $d = 1$ mm: ▽, $\gamma/\rho_{pf} = 5.88 \cdot 10^{-4}$; □, $\gamma/\rho_{pf} = 1.99 \cdot 10^{-3}$; +, $\gamma/\rho_{pf} = 2.94 \cdot 10^{-3}$.

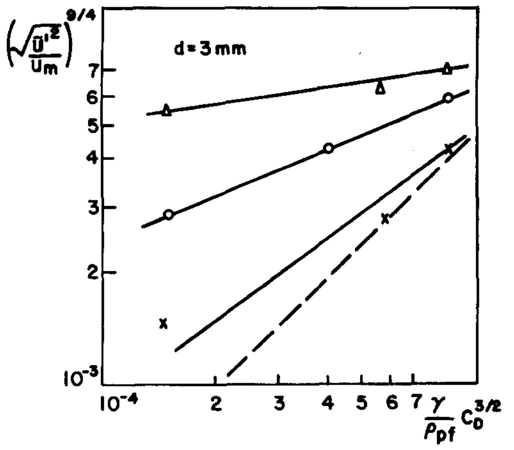

Figure 14. Dependence of the carrier fluid velocity fluctuations on the complex $(\gamma/\rho_{pf})c_D^{3/2}$ for various distances from the axis (pipe flow); u_m is the velocity at the pipe axis. Experimental data by Tsuji *et al.* (1984): ×, $y/R = 0$; ○, $y/R = 0.532$; △, $y/R = 0.937$; ---, the 4/9-power law.

is a suppression of turbulence. The latter is related to an interplay of the inertial properties of the admixture (leading to suppression of the gradient-born turbulence) and a decrease in the turbulence intensity in the near-wall region.

The present theory accounts for the above effects, as is seen in figure 14 where a comparison with the experimental data for the flow in vertical pipe is shown. In figure 14 the dependence $\sqrt{\overline{u'^2}}/u_m$ on $[(\gamma/\rho_{pf})c_D^{3/2}]$ for various distances from the pipe axis is plotted. The dashed line represents the 4/9-power law. It is seen that the experimental data corresponding to the axis of the flow does group near the curve corresponding to the 4/9-power law, whereas those corresponding to the regions further from the axis do not. The latter manifests itself clearly as the particle sizes and turbulence intensity modulation decrease (figure 15). The effect of the total mass content of the admixture is distinct for fine and coarse particles. For the former, an increase in γ leads to a decrease in the turbulence intensity as a result of the growth of the inertia of admixture; whereas for the latter particles, an increase in the total mass content of the admixture leads to an increase in the turbulence intensity. This results in an increase in the number of particle wakes per unit volume of the mixture and a corresponding growth of the average turbulence intensity.

The quantitative agreement between the theoretical 4/9-power and the experimental results is of considerable theoretical and practical interest. What remains to be done is to determine the empirical constant Ω in [35] and [36].

In figures 16–18 the results of the calculations of the fluctuations intensity for pipe flow and homogeneous turbulence are presented, and compared with the experimental data on turbulence intensity.† It is seen that for an appropriate value of the empirical constant Ω ($\Omega = 0.36$ for homogeneous turbulence and $\Omega = 0.6$ for pipe flow), fairly good agreement between the theory and experiments is achieved. This supports the supposition that the physical mechanism of wake and vortices shedding is the main turbulence generation process in flows with coarse particles.

Note that the empirical constant Ω is related to the ratio of the turbulence constants c and c_1. The above-mentioned values of Ω correspond to $c_1/c = 0.7$. An increase in c_1/c leads to a decrease in the value of Ω. The calculations show that in the proximity of the found value of the ratio c_1/c ($c_1/c \simeq 0.7$), the dependence of Ω on c_1/c is rather weak. It is emphasized that in the range of reasonable variation of the coefficients c and c_1, the value of Ω is of the order of unity.

†To calculate the relative velocity in the pipe flow, [53] is used.

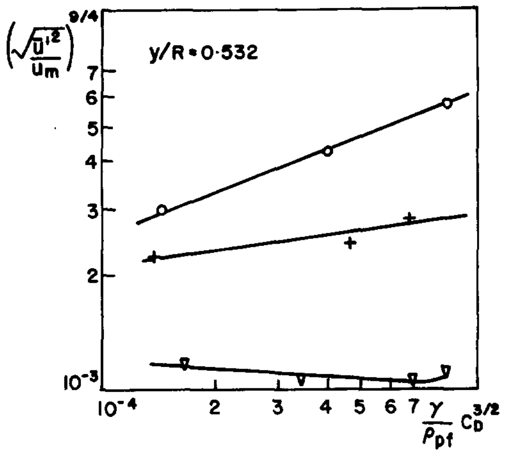

Figure 15. Dependence of the carrier fluid velocity fluctuations on the complex $(\gamma/\rho_{fp})c_D^{3/2}$ for various particle sizes; u_m is velocity at the pipe axis. Experimental data by Tsuji *et al.* (1984): ○, $d = 3$ mm; +, $d = 1$ mm; ▽, $d = 0.5$ mm.

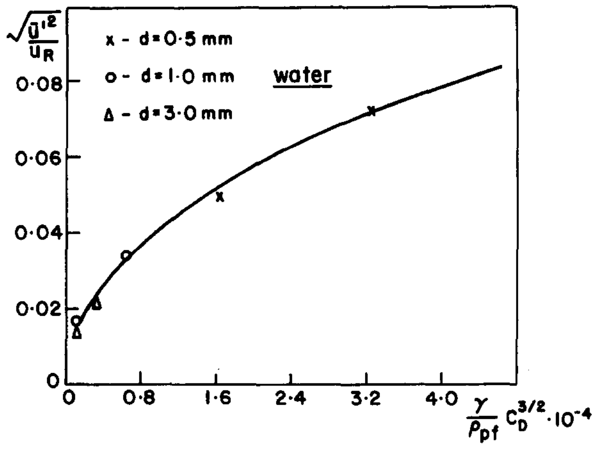

Figure 16. Dependence of the carrier fluid fluctuations on the complex $(\gamma/\rho_{pf})c_D^{3/2}$ (homogeneous turbulence; $\Omega = 0.36$, u_R is the relative velocity). Experimental data by Parthasarathy & Faeth (1990); ——, results of the calculations.

CONCLUSION

The proposed model of the particle–turbulence interaction describes a number of very important phenomena of the turbulence in particle-laden flows. It is emphasized that the applicability of this model is not restricted by the particle sizes, since it takes into account both the suppression of turbulence due to fine particles and its enhancement due to coarse ones. Therefore, various types of particle-laden flows are described by the proposed model and rather good agreement with experimental data is observed.

The following results have been obtained:

1. The proposed model describes quantitatively the process of suppression and generation (enhancement) of turbulence in particle-laden flows.
2. It is shown that the intensity of the velocity fluctuations is determined by four parameters: the total mass content of the admixture; the phase density ratio; the particle Reynolds numbers; and the particle size/turbulence scale ratio. In the two limiting cases of very small or very large particles, the turbulence intensity is determined solely by the total mass content or the total content/phase density ratio, respectively.

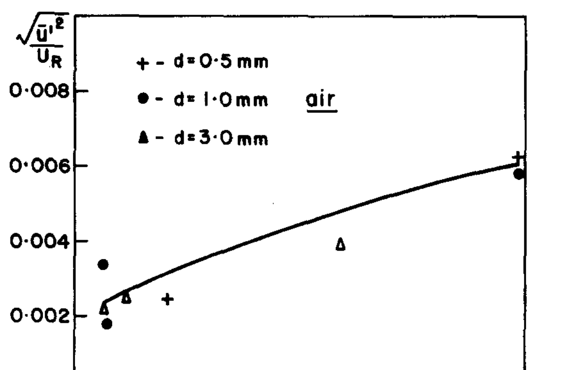

Figure 17. Dependence of the carrier fluid fluctuations on the complex $(\gamma/\rho_{pf})c_D^{3/2}$ (homogeneous turbulence; $\Omega = 0.36$, u_R is the relative velocity). Experimental data by Mizukami *et al.* (1992); ——, results of the calculations.

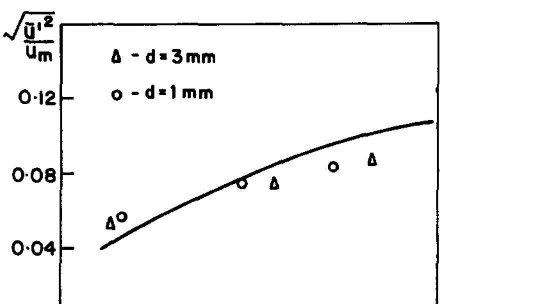

Figure 18. Dependence of the carrier fluid fluctuations on the complex $(\gamma/\rho_{pf})c_D^{3/2}$ (pipe flow; $\Omega = 0.6$, $c_D = 0.4$, u_m is the velocity at the pipe axis). Experimental data by Tsuji *et al.* (1984); ——, results of the calculations (pipe axis).

3. It is shown that in two-phase flows with coarse particles the logarithm of the modulated turbulence intensity in the particle wakes is proportional to the ratio $(\gamma/\rho_{pf})c_D^{3/2}$, with a coefficient of proportionality equal to 4/9. This is the 4/9-power law.
4. The particles–turbulence interaction is studied as a function of the nondimensional groups of parameters. The features of the effect of the velocity gradient and the turbulent wakes (behind the particles) on the turbulence intensity are investigated as a function of the particle sizes, the phase density ratio, the particle Reynolds numbers and the total mass content.
5. Comparison of the theoretical predictions with the experimental data for homogeneous turbulence, turbulent jet and pipe flows supports the 4/9-power law.
6. The effect of the particle-size distribution on the energy of the fluctuations in the particle wakes was estimated.
7. The effect of gravity on the energy of the fluctuations in the particle wakes was estimated.

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